This exam comprises 3 problems. The total number of points is 36. The final grade is obtained by dividing the number of points by 3,6.

Don't forget to write your name and student number.

Problem 1 (12 pts)
A thermal reactor contains $m = 150$ tonnes of natural uranium containing 0.72% of U-235 and runs with a neutron flux $\phi = 10^{13}\ cm^{-2}s^{-1}$.
The microscopic fission and neutron capture cross sections for U-235 are $\sigma_f = 579\ b$ and $\sigma_{n,\gamma} = 101\ b$, respectively.

a. Calculate the number of U-235 atoms in the fuel. (4 pts)
b. Determine the average power P in MW. (2 pts)
c. Calculate the absorption rate of U-235 (in s^{-1}). (4 pts)
d. Calculate the number of U-235 atoms consumed in a year and determine what percentage of the initial U-235 was consumed in a year. (2 pts)

Solution

a. Natural uranium has an isotopic composition of 0.72% of U-235 and 99,28% of U-238 (and 0.005% of other isotopes, which we neglect here). The number of U-235 atoms in the fuel is

$$N = m\frac{N_A}{M} = 150\ 10^6 \times 0.0072 \times \frac{6.022 \times 10^{23}}{0.0072 \times 235 + 0{,}9928 \times 238} = 2.73\ 10^{27}\ atoms$$

b. The power per cm^3 is given by

$$P = E_f R_f = E_f \Sigma_f \phi = E_f \sigma_f N \phi$$

The power is then

$$P = E_f \sigma_f N \phi = 200 \times 1.602\ 10^{-13} \times 579\ 10^{-24} \times 2.73\ 10^{27} \times 10^{13} = 506\ MW$$

c. The absorption rate of U-235 is due to both fission and neutron capture. The absorption cross section is

$$\sigma_a = \sigma_f + \sigma_{n,\gamma} = 579 + 101 = 680\ b = 680\ 10^{-24}\ cm^2.$$

The absorption rate of U-235 is given by

$$R_a = N\sigma_a\phi = 2.73\ 10^{27} \times 680\ 10^{-24} \times 10^{13} = 1.86\ 10^{19}\ s^{-1}$$

d. The number of U-235 atoms consumed in a year is

$$N\sigma_a\phi\ (365 \times 24 \times 3600) = 1.86\ 10^{19}(365 \times 24 \times 3600) = 5.85\ 10^{26}\ atoms$$

The initial load of U-235 is $2.73\ 10^{27}\ atoms$, and $5.85\ 10^{26}/2.73\ 10^{27} = 0.21$, that is 21% of the U-235 has been consumed in a year.

Problem 2 (12 pts)

Consider a critical fast reactor consisting of a homogeneous mixture of ^{239}Pu (fuel) and sodium (Na) made in a spherical shape. The (atom) number densities are $N_F = 0.00395 \times 10^{24}$ for plutonium and $N_S = 0.0234 \times 10^{24}$ for sodium.

Cross sections (in barn), average number of neutrons per fission and reproduction factors η are shown in the table below.

Element or Isotope	σ_γ	σ_f	σ_a	σ_{tr}	ν	η
Na	0.0008	0	0.0008	3.3		
Al	0.002	0	0.002	3.1		
Fe	0.006	0	0.006	2.7	—	—
^{235}U	0.25	1.4	1.65	6.8	2.6	2.2
^{238}U	0.16	0.095	0.255	6.9	2.6	0.97
^{239}P	0.26	1.85	2.11	6.8	2.98	2.61

a. Show that, for a critical reactor, the buckling can be written as (2 pts)

$$B^2 = \frac{k_\infty - 1}{L^2}$$

b. Find the expression of the radius of the spherical reactor as a function of k_∞ and L^2. (2 pts)

c. The reactor has a fast fission factor $\varepsilon = 1.04$ and a resonance escape probability $p = 0.96$. Calculate the infinite multiplication factor k_∞ and the diffusion area L^2. (4 pts)

d. Calculate the radius of the spherical reactor. (2 pts)

e. Calculate the extrapolation distance d and the radius of the bare spherical reactor. (2 pts)

Solution

a. The expression of the multiplication factor k is

$$k = \frac{\nu\Sigma_f}{\Sigma_a + DB^2}$$

For a critical reactor, we obtain

$$1 = \frac{\nu\Sigma_f}{\Sigma_a + DB^2} \Longrightarrow \Sigma_a + DB^2 = \nu\Sigma_f \Longrightarrow B^2 = \frac{\nu\Sigma_f - \Sigma_a}{D}$$

Dividing the numerator and denominator by Σ_a, we obtain

$$B^2 = \frac{\frac{\nu\Sigma_f}{\Sigma_a} - \frac{\Sigma_a}{\Sigma_a}}{D/\Sigma_a}$$

Using the expressions of $k_\infty = \nu\Sigma_f/\Sigma_a$ and of the diffusion area $L^2 = D/\Sigma_a$, we have

$$B^2 = \frac{k_\infty - 1}{L^2}$$

b. The buckling for a sphere is

$$B^2 = \left(\frac{\pi}{R}\right)^2$$

Using the previous result, we can write

$$B^2 = \left(\frac{\pi}{R}\right)^2 = \frac{k_\infty - 1}{L^2}$$

Therefore, the radius is

$$R = \pi\sqrt{\frac{L^2}{k_\infty - 1}}$$

c. The four factor formula is

$$k_\infty = \varepsilon p f \eta$$

ε, p and η are given and we just need to calculate the utilization factor f.

$$f = \frac{\Sigma_a^{fuel}}{\Sigma_a^{total}}$$

Using the data from the table, we have

$$\Sigma_a^{fuel} = N_F \times \sigma_a = 0.00395 \times 10^{24} \times 2.11 \times 10^{-24} = 0.00833\ cm^{-1}$$

$$\Sigma_a^{sodium} = N_S \times \sigma_a = 0.0234 \times 0.0008 = 0.000019\ cm^{-1}$$

$$\Sigma_a^{total} = \Sigma_a^{fuel} + \Sigma_a^{sodium} = 0.00835\ cm^{-1}$$

The utilization factor is then

$$f = \frac{0.00833}{0.00835} = 0.997 \simeq 1$$

With $\eta = 2.61$, the infinite multiplication factor is

$$k_\infty \simeq 1.04 \times 0.96 \times 1 \times 2.61 \simeq 2.61$$

We now need to calculate $L^2 = D/\Sigma_a$. The diffusion coefficient D is $1/3\Sigma_{tr}$.

$$\Sigma_{tr} = N_F \times \sigma_{tr}^{Pu} + N_S \times \sigma_{tr}^{Na}$$

$$\Sigma_{tr} = 0.00395 \times 6.8 + 0.0234 \times 3.3 = 0.104\ cm^{-1}$$

The diffusion coefficient is

$$D = \frac{1}{3\Sigma_{tr}} = \frac{1}{3 \times 0.104} = 3.21\ cm$$

Finally,

$$L^2 = \frac{D}{\Sigma_a^{total}} = \frac{3.21}{0.00835} = 384.4\ cm^2$$

d. The radius of the spherical reactor is

$$R = \pi \sqrt{\frac{L^2}{k_\infty - 1}} = \pi \sqrt{\frac{384.4}{2.61 - 1}} = 48.5\ cm$$

e. The extrapolation distance is

$$d = 2.13\ D = 2.13 \times 3.21 = 6.84\ cm$$

The radius of the bare spherical reactor is then

$$R_{bare} = R - d = 48.5 - 6.84 = 41.7\ cm$$

Problem 3 (12 pts)

Answer the following questions briefly (1 to 2 sentences).

a. What is the effect of delayed neutrons in a reactor? (2 pts)
b. Consider a thermal reactor. Give an example of a positive reactivity insertion. (2 pts)
c. Why is enriched fuel used in light-water reactor? (2 pts)
d. What is the purpose of a moderator in a thermal reactor? (2 pts)
e. Each fission of U-235 is accompanied by the emission of 2 to 3 neutrons. Describe what happens to the neutrons that do not cause a fission. (2 pts)
f. Xenon-135 is a neutron poison created from fission reactions. What is the effect of the accumulation of these poisons on the reactivity? (2 pts)
g. Explain why a reactor is initially fuelled with more than the minimum amount of fuel necessary for criticality.

Solution

a. The delayed neutrons have the effect of increasing the effective lifetime of the neutrons.
b. An example of a positive reactivity insertion is the (partial) removal of control rods in the reactor.
c. Since water has a non-negligible neutron absorption cross section, the neutrons absorbed do not contribute to fission reactions. To compensate for this loss of neutrons, the amount of fissile material (U-235) needs to be increased.
d. The purpose of the moderator is to slow down the neutrons to thermal energies.
e. Neutrons that don't cause fissions can be absorbed in the fuel in a (n, γ) capture reaction, can be absorbed in the material of the reactor, can be absorbed in water (moderator), creating deuterium (H-2) or can leak out from the reactor.

f. The neutron poison Xe-135 inserts a negative reactivity, because of its high neutron absorption cross section.
g. If it were fuelled with only its minimum critical mass, the reactor would fall subcritical after the first fission.

Avogadro number $N_A = 6.022 \times 10^{23} mol^{-1}$

Cross section $1\ b = 10^{-24} cm^2$

Conversion factors $1\ MeV = 1.602\ 10^{-13}\ J$ and $1\ W = 1\ J/s$

Number of nuclei:

$$N = \frac{m\ N_A}{M}$$

Activity (Bq):

$$A = \lambda N = A_0 e^{-\lambda t}$$

Multiplication factor:

$$k = \frac{\nu \Sigma_f}{\Sigma_a + DB^2}$$

$$k_\infty = \frac{\nu \Sigma_f}{\Sigma_a}$$

Diffusion area:

$$L^2 = \frac{D}{\Sigma_a}$$

Diffusion coefficient:

$$D = \frac{1}{3\Sigma_{tr}}$$

With Σ_{tr} the macroscopic transport cross section.

Extrapolation distance: $d = 2.13\ D$

Reactivity:

$$\rho = \frac{k-1}{k}$$

Total average power of a reactor

$$P = E_f \times R_f$$

Rate of reaction of type i

$$R_i = \Sigma_i \phi = N \sigma_i \phi$$

Four factor formula

$$k_\infty = \varepsilon p f \eta$$

Thermal utilization factor

$$f = \frac{\Sigma_a^{fuel}}{\Sigma_a^{total}}$$

Buckling and neutron flux for different reactor geometries

Geometry	Dimensions	Buckling B^2	Flux	A
Infinite slab	Thickness a	$\left(\frac{\pi}{a}\right)^2$	$A\cos\left(\frac{\pi}{a}x\right)$	$1.57 \times \frac{P}{a\,E_f\,\Sigma_f}$
Rectangular parallepiped	a x b x c	$\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2$	$A\cos\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{b}y\right)\cos\left(\frac{\pi}{c}z\right)$	$3.87 \times \frac{P}{V\,E_f\,\Sigma_f}$
Infinite cylinder	Radius R	$\left(\frac{2.405}{R}\right)^2$	$AJ_0\left(\frac{2.405\,r}{R}\right)$	$0.738 \times \frac{P}{R^2 E_f \Sigma_f}$
Finite cylinder	Radius R, height H	$\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$	$AJ_0\left(\frac{2.405\,r}{R}\right)\cos\left(\frac{\pi\,z}{H}\right)$	$3.63 \times \frac{P}{V\,E_f\,\Sigma_f}$
Sphere	Radius R	$\left(\frac{\pi}{R}\right)^2$	$A\,\frac{1}{r}\sin\left(\frac{\pi\,r}{R}\right)$	$\frac{P}{4R^2\,E_f\,\Sigma_f}$